Previous chapter: mag fields interact w/ moving charges

$$\vec{F} = q\vec{v} \times \vec{B}$$

Currents: moving charges $\therefore$ $\vec{B}$ interacts w/ currents

$$\vec{F} = I\vec{L} \times \vec{B}$$

1820 Hans Oersted showed that currents generate B-fields that move compass needles

$\Rightarrow$ this was probably 1st thing to connect electricity w/ magnetism $\Rightarrow$ electromagnetism

so if currents of moving charges generates B fields then a single charge must generate field

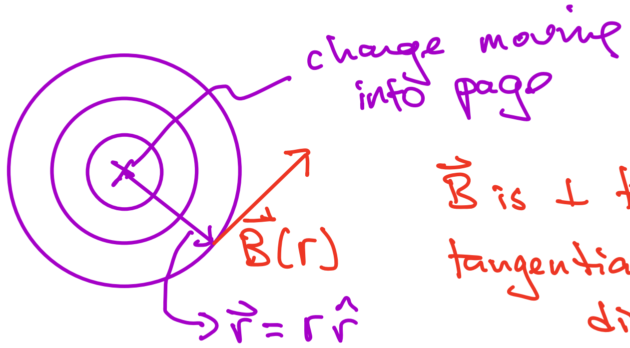
$$\vec{B}(r) = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$$

$\mu_0 = 4\pi \times 10^{-7}$ T/A tesla/amp

$\vec{v}$ velocity of moving charge q

r distance from q to point of B field

$\hat{r}$ unit direction from q to " " " "



$\vec{B}$ is $\perp$ to $\vec{v}$ & $\vec{r}$ so is tangential to circle around $\vec{v}$ direction

Field due to current in wire

each charge in current generates $\vec{B}$

$$\vec{B}_{tot} = \sum_{i=1}^{\#\text{charges}} \vec{B}_i \Rightarrow \int d\vec{B}$$

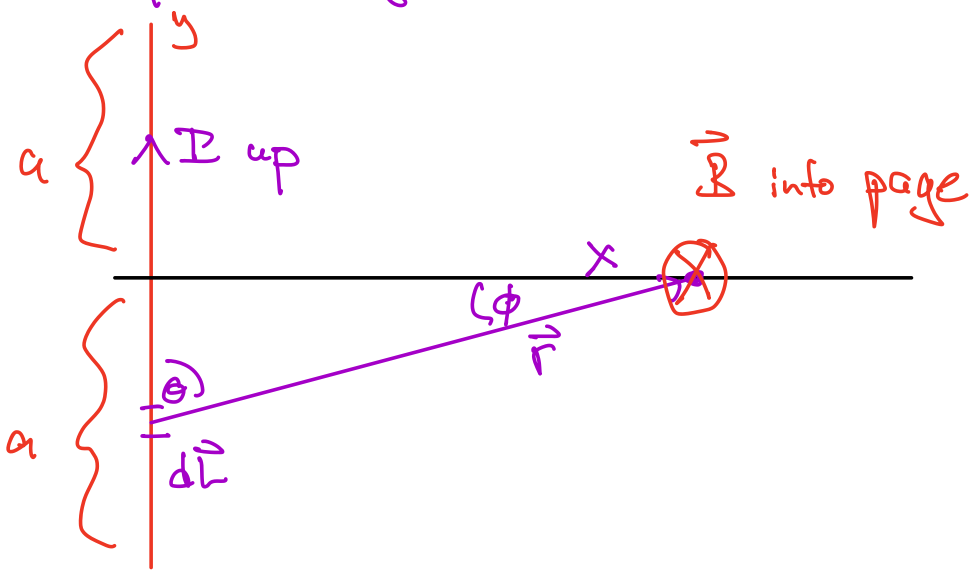
write $d\vec{B} = \frac{\mu_0}{4\pi} dq \frac{\vec{v} \times \hat{r}}{r^2}$ $\quad dq$ = tiny bit of charge

$$\vec{v} = \frac{d\vec{L}}{dt} \text{ and } dq\,\vec{v} = dq \cdot \frac{d\vec{L}}{dt} = \frac{dq}{dt} d\vec{L} = I\,d\vec{L}$$

so $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\,d\vec{L} \times \hat{r}}{r^2}$ $\quad$ Biot-Savart Law

integrate over wire

ex: $\vec{B}$ from straight wire



$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{L} \times \hat{r}}{r^2} \qquad d\vec{L} \times \hat{r} = dL \sin\theta \ \ \& \ \ dL = dy$$

note $\theta + \phi = 90^0 \therefore \sin\theta = \cos\phi$

and $\cos\phi = x/r$ or $r = x/\cos\phi = x\sec\phi$

also $\tan\phi = y/x$ so $dy = x\sec^2\phi\, d\phi$

transform integral to

$$\frac{\mu_0}{4\pi} I \frac{\overbrace{(x\sec^2\phi\, d\phi)}^{dL} \cdot \overbrace{\cos\phi}^{\sin\theta}}{x^2\sec^2\phi}$$

$$B = \int_{-\phi_0}^{\phi_0} \frac{\mu_0 I}{4\pi x} \cos\phi\, d\phi \qquad \text{where } \tan\phi_0 = \frac{a}{x}$$

$$= \frac{\mu_0 I}{4\pi x} \sin\phi \Big|_{-\phi_0}^{\phi_0} = \frac{\mu_0 I}{2\pi x} \sin\phi_0$$

$$\sin\phi_0 = \frac{a}{\sqrt{a^2+x^2}}$$

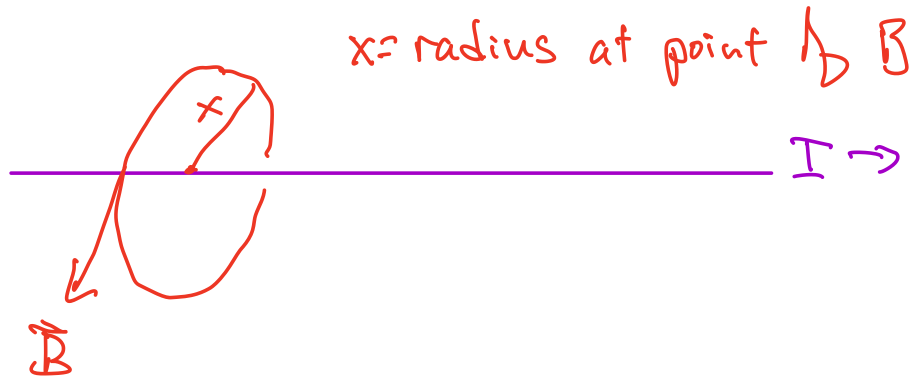
$$\boxed{B = \frac{\mu_0 I}{2\pi} \frac{a}{x\sqrt{a^2+x^2}}}$$

now take limit as $a \to \infty$, wire becomes "infinite"

$\frac{a}{\sqrt{a^2+x^2}} \to 1$ so $B \to \frac{\mu_0 I}{2\pi x}$

$$\boxed{B = \frac{\mu_0 I}{2\pi x}}$$

x = dist from wire



Rules for getting B-field from current in wire

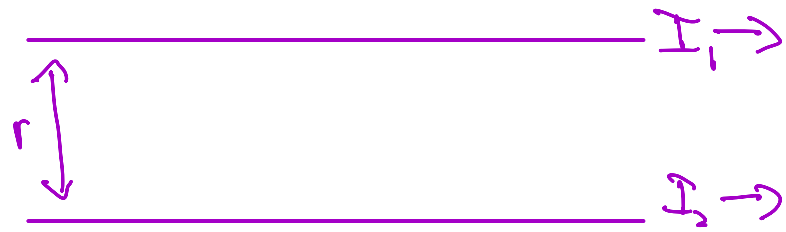
1. magnitude of B is $B = \frac{\mu_0 I}{2\pi r}$

   r = distance from wire

2. B loops around wire

3. use RHR → thumb along current, fingers point along B

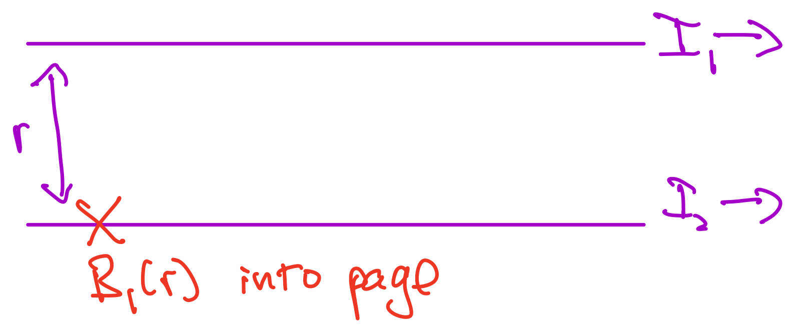
Force on 2 wires carrying current



wire 1 makes $\vec{B}_1$ field at wire 2

$$B_1(r) = \frac{\mu_0 I}{2\pi r}$$

by RHR $B_1(r)$ is into page



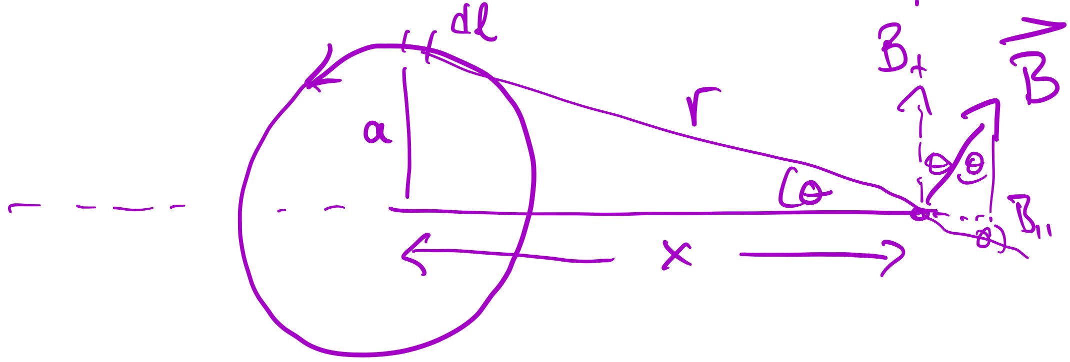
$\vec{F}_2$ = force on wire 2 from B from wire 1

$\vec{F}_2 = I_2 \vec{L}_2 \times \vec{B}_1 \Rightarrow F_2$ is up towards wire 1

$$F_2 = I_2 L_2 B_1 = I_2 L_2 \frac{\mu_0 I_1}{2\pi r} = L_2 \cdot \frac{\mu_0 I_1 I_2}{2\pi r}$$

force per length $\dfrac{F_2}{L_2} = \dfrac{\mu_0 I_1 I_2}{2\pi r}$

B field from circular current loop



$B_\perp$ cancels out by symmetry

$$dB_{\parallel} = \frac{\mu_0 I}{4\pi} \frac{dl \sin\theta}{x^2+a^2}$$

integrate around loop → everything is constant

$$B_{\parallel} = \int \frac{\mu_0 I}{4\pi} \frac{\sin\theta}{x^2+a^2} \, dl$$

$$= \frac{\mu_0 I}{4\pi} \frac{\sin\theta}{x^2+a^2} \overbrace{\int dl}^{2\pi a}$$

$$\sin\theta = \frac{a}{\sqrt{x^2+a^2}}$$

so $B = \dfrac{\mu_0 I \quad a^2}{2(x^2+a^2)^{3/2}}$ along axis

note: near center, $x \ll a$, $B = \dfrac{\mu_0 I}{2a}$ constant!

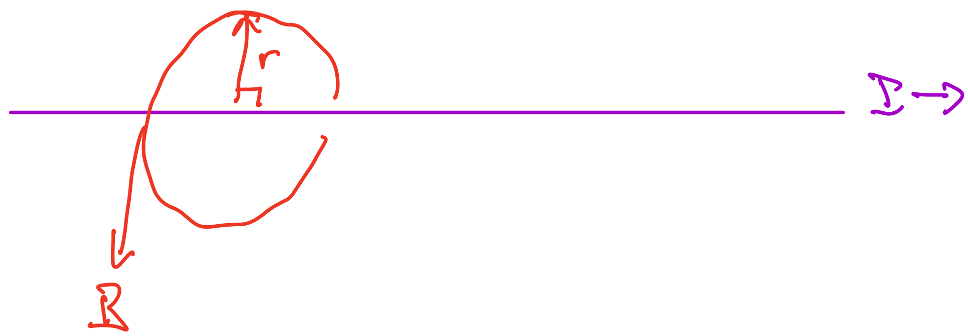
if we have a coil w/ N turns,
each one contributes

$$\Rightarrow B_{tot} = NB_1 = \frac{\mu_0 N I a^2}{(x^2+a^2)^{3/2}}$$

use RHR → curl fingers along I,
B points in dir of thumb away

this is called a "dipole"

Back to B from infinite wire



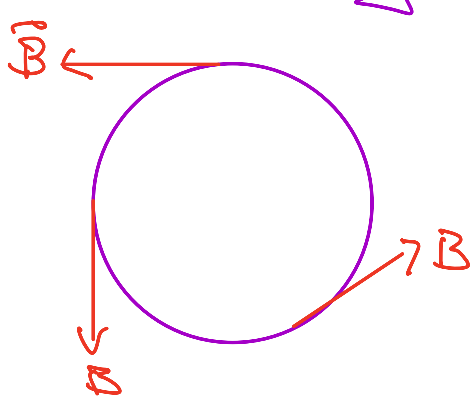
$B = \frac{\mu_0 I}{2\pi r}$ circling around wire using RHR

note: $2\pi r$ = circumference of circle w/ radius r

so $B \cdot 2\pi r = \mu_0 I$

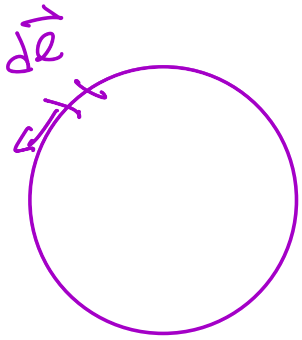
if we draw an imaginary circle around the wire, then I is the current inside the circle

B is constant in magnitude but <u>not</u> direction



in calculus we can integrate around a closed path

$\Rightarrow$ define $d\vec{l}$ = differential length, direction is along path



then $\oint \vec{B}\cdot d\vec{l}$ is integral of $\vec{B}$ around path

for long wire $\oint \vec{B}\cdot d\vec{l} = \vec{B}\cdot\oint d\vec{l}$

$\Rightarrow$ since $\vec{B}$ is parallel to $d\vec{l}$:

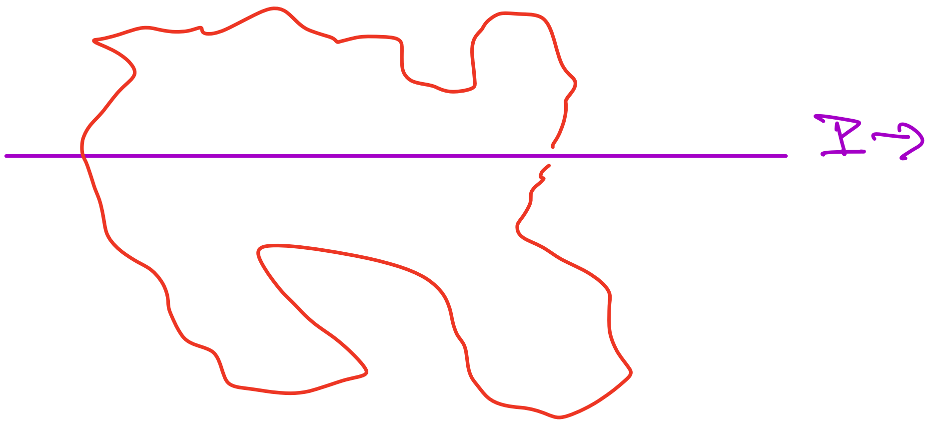
$$\vec{B}\cdot\oint d\vec{l} = B\oint dl = B\cdot 2\pi r$$

and I = current inside loop

$$\oint_{\text{closed path}} \vec{B}\cdot d\vec{l} = \mu_0 I_{\substack{\text{inside}\\ \text{path}}}$$

this is Ampere's law and is true in general!

ex:



- draw any closed path around wire
- integrate $\vec{B}$ around that path

$$\oint_{\text{path}} \vec{B} \cdot d\vec{\ell} = \mu_0 I_{inside} = \mu_0 I$$

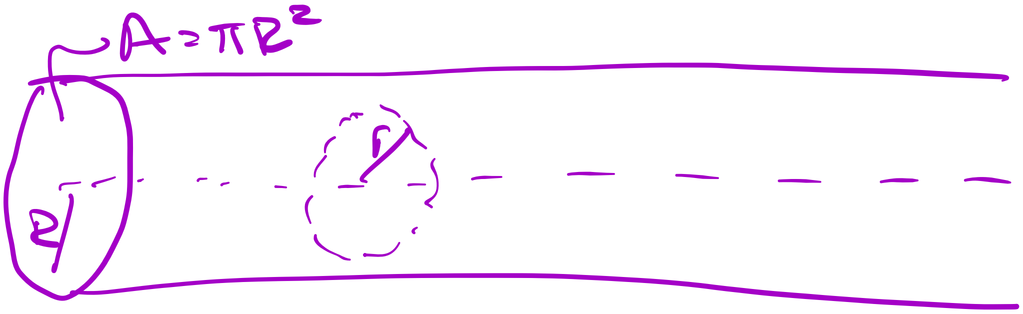
BUT that's not useful really!

Ampere's law is most useful when B is constant along path so that

$$\oint_{\text{path}} \vec{B} \cdot d\vec{\ell} = B \cdot \underset{\text{path length}}{L}$$

ex: what is $\vec{B}$ inside a wire?

ex: B inside conductor



current density $J = I/\pi R^2 = \text{constant}$

now draw loop with radius $r < R$ inside wire

$\Rightarrow$ we expect $\vec{B}$ is also constant at constant r, and points along circle just like for $r > R$

$\Rightarrow$ current inside loop: $I(r) = J \cdot \pi r^2$

so $\oint \vec{B} \cdot d\vec{l} = B \cdot 2\pi r = \mu_0 I(r) = \mu_0 J \pi r^2$

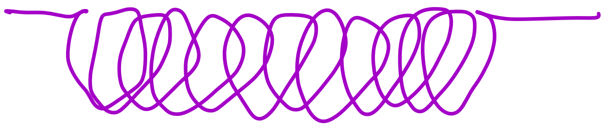
$J = I/\pi R^2$ so

$$B \cdot 2\pi r = \mu_0 I \frac{r^2}{R^2}$$

$$\boxed{B = \frac{\mu_0 I r}{2\pi R^2}}$$

when $r = R$, $B = \frac{\mu_0 I}{2\pi R}$

Solenoid: loops of wire, cylindrical



⇒ each "turn" of wire is a current loop that generates a B-field

⇒ $\vec{B}$ from each loop adds as a vector

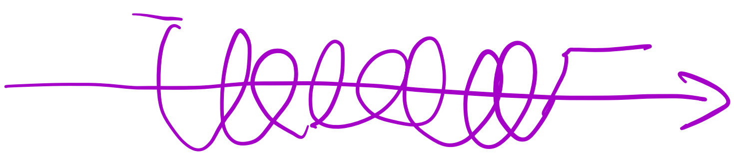
Solenoid

direction of current in the coil

current into page

current out of page

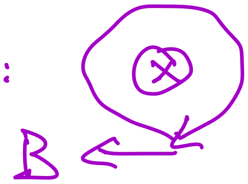
can think of solenoid as having N coils
near center of each loop B = const
so net $\vec{B}$ in a solenoid is constant down axis



- can show that B is ~ constant inside
- " " " B is ~ 0 outside coil

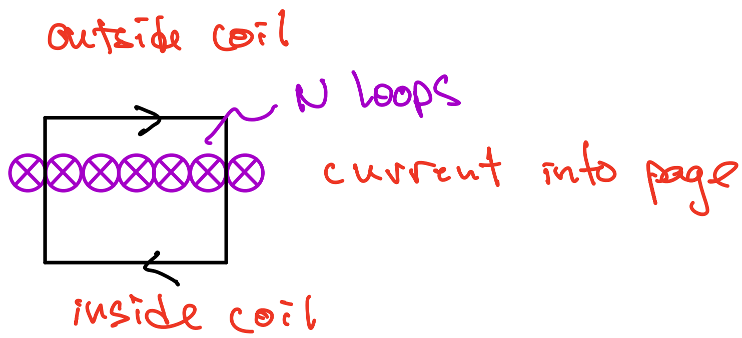
Find B inside solenoid!

B from each coil:



by RHR

invent path like this



use Ampere's law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{in}$

$$I_{in} = NI$$

- along vertical legs $\vec{B} \perp d\vec{l}$ so $\vec{B} \cdot d\vec{l} = 0$
- along horizontal leg inside coil:

$$\int \vec{B} \cdot d\vec{l} = B \cdot L = \mu_0 NI$$

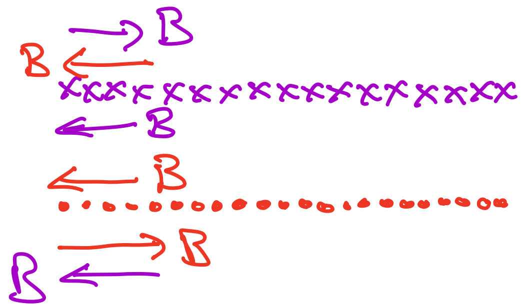
so $B = \mu_0 \frac{N}{L} I$

let $n = \frac{N}{L}$ for full solenoid

then $B = \mu_0 n I$ inside

note: opposite side of coil has same current of course

$\Rightarrow$ B is from entire loops!

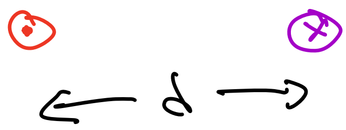


outside coil, B from each side will cancel

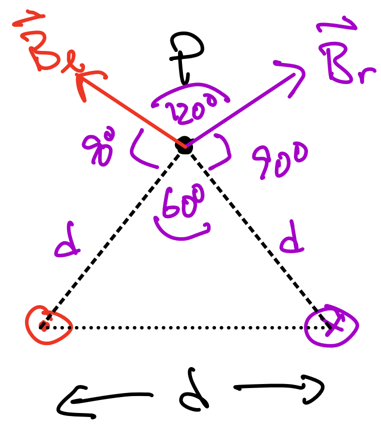
so $B \sim 0$ outside coil

$B = \mu_0 n I$ inside coil w/ current I

2 wires w/equal & opposite currents I
separated by distance d

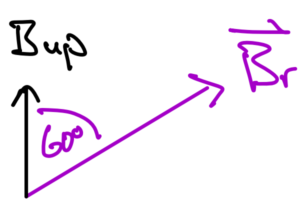


calculate $\vec{B}$ (dir & magnitude at position P
that forms isosceles triangle



⇒ field due to right current ⊗ adds as vector
to field due to left current

⇒ components pointing up will add
⇒ " " left & right will cancel



$B_r = \frac{\mu_0 I}{2\pi d}$ $\quad$ $B_{up} = B_r \cos 60^0 = \frac{B_r}{2}$

total B_{up} ⇒ contibutions from B_r & B_ℓ

$B_{tot} = 2 B_{up} = B_r = \frac{\mu_0 I}{2\pi d}$